Evidence for Excellence in Education

Report

Developing National Curriculum-based Learning Progressions

Mathematics

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Developing National Curriculum-based Learning Progressions

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Contents

Preface

NFER is both a charity and a specialist research organisation. We exist to make a positive difference to learners.

We continue to be recognised as a leading research and assessment organisation throughout the UK, providing independent evidence to improve education and learning in children and young people's lives. Key stakeholders, research clients and headteachers described NFER as: reliable, rigorous, professional, expert and independent in market research conducted in 2012. Our influence with schools and children continues to grow; in 2013 over 2.5 million school children took tests developed by us and over 50 per cent of schools in England took part in research conducted by us. We also combine our own evidence with that of others to offer an unbiased expert voice.

Through expert research and our extensive knowledge of education and assessment, we offer a unique perspective on today's and tomorrow's educational challenges. Our insights are relevant, accessible and inform policy and practice across the world, and our evidence is cited in government reports, academic journals and the wider media. Successive governments have used it to inform policy thinking.

We develop assessments to the highest technical and educational standards for test publishers and government bodies. During 2013/14, we continued developing and trialling National Curriculum assessments with the Standards and Testing Agency in England and, in Wales, our work continued supporting education reform as the developers and suppliers of the new national reading and numeracy tests.

NFER's assessment expertise rests on a solid foundation of research, ranging from piloting and evaluation of new assessment models, to research into effective test development, marking, administration, analysis and reporting methodologies.

We have a team of over 80 highly qualified professional researchers, statisticians and assessment experts who offer extensive experience to ensure our assessments are reliable, fair and informative. NFER's Research and Product Operations department is equally highly skilled in developing and managing large scale and complex research and assessment projects across the education sector.

1 Introduction

At the end of 2013, NFER was commissioned by Renaissance Learning to develop *learning progressions* in mathematics and reading based on the 2014 National Curriculum in England. The *learning progressions* were subsequently mapped to Renaissance Learning's STAR Maths programme.

This report describes the development processes adopted to ensure the *learning progressions* were appropriate for use and would support the delivery of the programmes of study for mathematics at key stages 1 to 4 and the assessment of pupil progress. A parallel report is available for reading.

1.1 Background: National Curriculum reform in England

The beginning of the 2014/15 academic year saw the launch of the new National Curriculum, to be taught in years 1–11 in key stages 1–4. The new curriculum arose as a result of a comprehensive review published in December 2011 (DfE, 2011). Overall it is clear that the demand in the new curriculum, especially in some subject areas, has been explicitly increased and that specific skills and aspects of knowledge will be required to be taught at an earlier age than previously.

Pupil attainment against the pre-2014 National Curriculum was measured by means of an eight-level scale. At each level, there were subject-specific descriptions of performance, against which achievement could be assessed. At the same time as the new curriculum was introduced (September 2014), this level-based assessment system was abolished.

The decision to remove the National Curriculum levels was one of the recommendations of the National Curriculum review. The review panel concluded that the National Curriculum level descriptors 'lacked precision' and that the way levels were being used was inhibiting progress. It was recommended that:

> *[…] the purpose of statutory assessment would change from assigning a 'best fit' level to each pupil to tracking which elements of the curriculum they have adequately achieved and those which require more attention.*
>
> **(DfE, 2011)**

The removal of National Curriculum levels has been heralded as a move to allow much greater autonomy for schools to develop their own assessment systems. However, alongside such autonomy, schools have the responsibility to ensure that pupils are progressing towards the expected end of key stage standards and to demonstrate evidence of such progress to Ofsted and other key stakeholders.

The new National Curriculum programmes of study for mathematics set out what is to be taught by the end of each academic year in key stages 1 and 2 and by the end of each key stage in key stages 3 and 4. The Department for Education have also published a consultation on end of key stage performance descriptors for key stages 1 and 2 (DfE, 2014). However, it is entirely up to schools to develop their own assessment systems to judge what pupils have learned or mastered and to track the attainment and progress of pupils within and across key stages. This will require teachers to develop a detailed understanding of the relationship between the new curriculum and expected standards of achievement and progress.

Learning progressions offer a means of supporting teachers in assessing progress against the new National Curriculum.

1.2 Learning Progressions

Although the National Curriculum programmes of study define what is to be taught, they do not always specify all the steps in learning that a pupil must master in order to progress. *Learning progressions* (sometimes referred to as learning trajectories or progress maps) detail the way in which knowledge and skills in a particular curriculum area generally develop. *Learning progressions* are not new – they have been used extensively in Canada, the US[1], Australia and New Zealand.

According to James Popham, a key advocate of *learning progressions*:

> *A learning progression is a carefully sequenced set of building blocks that students must master en route to mastering a more distant curricular aim. These building blocks consist of subskills and bodies of enabling knowledge.*
>
> **(Popham, 2007)**

Learning progressions are becoming increasingly well known in the UK and their use has been supported by educational experts (Heritage, 2008; Black, Wilson & Yao, 2011; Wiliam, 2014).

Learning progressions based on the National Curriculum break down the steps in learning that will lead to the achievement of key curricular targets, including the identification of the key 'building blocks' or *focus skills* that need to be mastered en route. Understanding and being familiar with *learning progressions* in mathematics will enable teachers to construct teaching plans that provide pupils with opportunities and experiences to acquire the knowledge and develop the skills or sub-skills that will lead to the achievement of the next curricular target. Identifying the *focus skills* (see section 4.2) also provides a framework for assessing progress by pinpointing what evidence of progress may usefully be collected and when. It will enable teachers to measure where pupils are at particular points in time and how likely they are to achieve the expected standard at the end of the relevant key stage.

In checking progress regularly against the *learning progressions*, particularly the embedded *focus skills*, teachers will be carrying out meaningful formative assessment and also collecting evidence of progress to share with parents and Ofsted.

[1] For a detailed description of the development of learning progressions for mathematics in the US see Renaissance Learning, 2013.

1.3 Structure of the report

The remainder of this report describes the processes involved in the analysis of the new National Curriculum in England in order to develop a set of *learning progressions* to support formative assessment of pupil progress against the programme of study for mathematics.

Chapter 2 outlines the overarching principles of the development process and chapter 3 focuses on the development of the framework or hierarchy within which the detailed *learning progressions* were created and organised. This is followed by a detailed description of the way in which the *learning progressions* were constructed (chapter 4), including the identification of gaps and inconsistencies within the programme of study for mathematics and how these were resolved. Other chapters detail the mapping of the *learning progressions* to Renaissance Learning's STAR assessment skills (chapter 5), the development of a teachable order for the *learning progressions* (chapter 6) and the production of teacher tables providing more detailed information for each of the *focus skills* (chapter 7).

2 Overarching development process

The development process spanned ten months and entailed a collaborative relationship among NFER's research team and Renaissance Learning. The process was necessarily iterative, requiring that rounds of development were followed up by detailed expert feedback and the material revised accordingly.

The development of the final product occurred in ten stages, which are presented as discrete stages here but, in practice, often functioned recursively to inform each other. After initial material generation and internal review, all stages underwent two further levels of review, first by NFER Research Associates (additional subject experts) and then by subject experts at Renaissance Learning. The stages are detailed briefly below and will be expanded in more detail in later chapters.

1. Familiarisation with the new National Curriculum.

2. Development of the underlying framework or hierarchy.

Nine discrete categories or *domains* which roughly align with topic sub-divisions in the National Curriculum were identified: place value, arithmetic, fractions, algebra, measurement, geometry, probability, ratio and proportion, and statistics. Each of these was broken down further into groups of *skill areas.*

3. Extracting the National Curriculum *statements.*

Individual skills required at each year of compulsory schooling were extracted from the National Curriculum and entered into a spreadsheet under a relevant *domain* and *skill area*; the whole mathematics curriculum (early years – year 11) was included.

4. Dissecting the National Curriculum statements to develop the *skill statements.*

The National Curriculum statements were distilled into discrete *skill statements* with each *skill statement* defining a skill that may form the focus of a lesson plan.

5. Establishing the *learning progressions* within each domain.

Framed by professional insight into classroom practices, the *skill statements* derived from the National Curriculum were chronologically presented to reflect a logical sequence in which the skills could be taught.

6. Revising and developing the learning progressions.

Additional *skill statements* were generated by subject experts to contextualise the atomised National Curriculum skills within a complete and realistic chronology of the way learning progresses in the classroom.

7. Identifying the focus skills.

For each domain, subject experts identified a number of *skill statements* as *focus skills*. Conceptualised as the skills upon which further learning is contingent, these skills would form the basis of the later generation of teacher table information.

8. Mapping the *learning progressions* to Renaissance Learning's STAR Math skills.

The *skill statements* contained in the *learning progressions* were mapped to Renaissance Learning's US progressions. Similarities between the two sets of skills were identified and individual components matched up. The purpose of this process was to identify the elements of the new National Curriculum for mathematics that could be assessed by Renaissance Learning's extensive bank of STAR mathematics items.

9. Developing a *teachable-order*

Subject experts manipulated the *skill statements* within the *learning progressions* to establish their optimal position in the chronology of each domain and, in the primary age range, within the academic year. The resulting *teachable order* outlines a rational sequence in which the skills could be taught.

10. Creating the *teacher tables.*

For each *focus skill,* a *teacher table* was created. The material generated from the *learning progressions* and *teachable order* was synthesised by subject experts who also provided additional supporting information, for example, terminology, concepts and suggestions for supporting pupils with English as an additional language (EAL).

3 Developing a hierarchy for the Learning Progressions

The Star Maths assessments required the mathematics National Curriculum to be categorised in a two level hierarchical structure of which the first level groupings are called *domains* and the second level groupings *skill areas.* The first of these was relatively straight forward to agree as the National Curriculum is already divided into clear strands. However, the strands are not consistently named across the years and key stages. Statements referring to addition, for example, are given under the strand *Number – addition and subtraction* in years 1 and 2, while in year 6 they are grouped under *Number – addition, subtraction*, *multiplication and division*. For key stage 3, the strand under which addition statements are located is even broader: *Number*. The priority in defining the *domains,* therefore, was to agree groupings which could be used consistently across the entire age range. In some cases, the National Curriculum strands were combined while in others, the strands were divided into smaller groupings. The development of the mathematics *hierarchy* was very much an iterative process and the *domains* were not decided at one single point in time. Rather, a draft *domain* structure was devised which was amended in light of the later developmental work devising the *skill areas* and *skill statements*.

The second level grouping within the *hierarchy* (the *skill areas*) required more consideration. The National Curriculum statements were examined across all years, one *domain* at a time. The aim of this examination was to discern the underlying structure and the relationships between the curriculum content in each *domain*. The result was that each of the draft *domains* was further broken up into smaller topic areas under which all the curriculum statements could be grouped. As mentioned previously, the iterative nature of the process meant that the *skill areas* were fine-tuned as the *skill statements* were subsequently developed, highlighting further details for consideration.

One notable difference between the primary and secondary curricula is the treatment of the *Working mathematically* skills. In key stages 1 and 2, *Working mathematically* is combined with the subject content statements. However, in key stages 3 and 4 the *Working mathematically* skills are presented separately to the subject content. Many of these skills overarch a large proportion of the curriculum and would be applicable to several *domains*. It was decided that the inclusion of the *Working mathematically* skills within the subject content would create an unmanageable number of repeated skills across the *domains* and *skill areas*. In order to preserve the emphasis placed on the *Working mathematically* skills, it was agreed that they would be linked to the subject content via the teacher tables.

As well as much internal revision, the draft *hierarchy* was also reviewed by additional external subject experts and by Renaissance Learning. The final *hierarchy* is shown in the tree diagram reproduced in Appendix A.

4 Developing Learning Progression skill statements from the NC Programme of Study

4.1 Creation of *skill statements*

Once the *hierarchy* had been devised, development of the *learning progressions* could begin for each of the identified *domains*. Due to their often broad nature, the statements within the National Curriculum were scrutinised and broken down into several shorter fragments, each of which could form the teaching basis of a lesson plan. For example, the year 5 National Curriculum statement *'recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number'* was broken down into three separate skill statements:

1) *'Recognise mixed numbers and improper fractions'*
2) *'Write mathematical statements greater than 1 as a mixed number'*
3) *'Convert between mixed numbers and improper fractions and vice versa'*

The National Curriculum includes non-statutory guidance for teachers. Additionally, there are several non-statutory documents associated with the National Curriculum, such as the GCSE maths specifications produced by the Awarding Bodies, which are relevant for key stage 4. These documents were consulted when determining the skill statements within each *domain,* along with the *US learning progressions* developed by Renaissance Learning.

The *learning progression* within each domain needs to consist of a set of *skill statements* outlining the progression of a particular skill or concept within mathematics. Two problems were encountered when building the *learning progressions* from the *skill statements* that had been developed by fragmenting National Curriculum statements. In some areas of the National Curriculum there were skills which were not explicitly mentioned in the statements but where it was considered essential to have or to understand them in order to progress onto the next skill within the area. In these instances, intermediary *skill statements* were developed to 'bridge the gap'. The second challenge was that some National Curriculum statements contain similar elements which could appear to repeat the same learning across the years. Where the fragmented National Curriculum statements seemed similar, differences had to be highlighted, for example by setting boundaries as to the size of the numbers the pupils should work with. An example of this can be seen in *Number – Arithmetic operations.* There are two identical statements in the National Curriculum for years 5 and 6 that require pupils to solve problems using addition and subtraction. Table 1 shows how boundaries were set according to guidance in the National Curriculum to ensure that the two statements showed progression between the years.

Table 1. Differentiation of the NC statements into *skill statements*

Year	National Curriculum statement	*Skill statements*
5	*solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why*	Solve addition and subtraction multi-step problems up to 100,000 in contexts, deciding which operations and methods to use and why
6	*solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why*	Solve addition and subtraction multi-step problems up to 1,000,000 in contexts, deciding which operations and methods to use and why

An additional issue was that some National Curriculum statements, while not identical, include material from earlier years. After initial fragmentation, some skill statements were recombined upon reflection, again to ensure that successive years developed the progression. National Curriculum statements appearing in lower years were sometimes more fragmented than statements from older years for this reason. The statements in Table 2 demonstrate this issue. The year 1 National Curriculum statement is fragmented into three *skill statements*. However the year 2 *skill statement* is almost identical to the National Curriculum statement because only the additional requirement to 'estimate' numbers sets it apart from the year 1 demand.

Table 2. Fragmentation of the NC statements into *skill statements*

Year	National Curriculum statement	*Skill statements*
1	*identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least*	Identify whole numbers from 0 to 100 using concrete representations such as objects and pictures (including the number line) Represent whole numbers from 0 to 100 using concrete representations such as objects and pictures (including the number line) Use the language of: equal to, more than, less than (fewer), most, least
2	*identify, represent and estimate numbers using different representations, including the number line*	Identify, represent and estimate whole numbers up to 100 using the number line and other representations

Whether they were directly fragmented from National Curriculum statements, introduced or amended to produce a seamless *learning progression,* all *skill statements* are aligned to a statement from the National Curriculum. An example of this relationship is given in Appendix B.

With the exception of G*eometry,* the *domains* do not extend from Reception to year 11. Some *domains* are associated mostly with the key stage 1 and key stage 2 age range. The *Measurement learning progression,* for example, extends from Reception through to year 6. By contrast, other domains such as *Probability* are primarily associated with the key stage 3 and key stage 4 National Curriculum.

Following the development of the *learning progressions,* there was a rigorous reviewing process within the NFER research team and by Renaissance Learning.

4.2 Identification of focus skills

Building upon the development of *learning progressions*, a subset of *focus skills* was identified. *Focus skills* are defined as skills that are essential to progression, that support the development of other skills in the same or future years and/or are central to the emphases of the National Curriculum. These skills constitute the areas that pupils need to master in order to become successful mathematicians.

To help with the identification of the *focus skills* in mathematics, the US *learning progressions* were interrogated to identify how *focus skills* had been assigned within different *skill areas*. This provided a guideline for the proportion of *learning progression skill statements* that could be designated as *focus skills* and helped shape the process to be adopted in England. The NC-based *learning progressions* were then scrutinised to identify the skills considered to be essential in underpinning future learning. As with previous developmental steps, final reviews were made by both the external subject expert Research Associates and Renaissance Learning

Out of nearly 1000 *learning progression skill statements* for mathematics, about 30 per cent were identified as *focus skills*. These *focus skills* were not equally distributed across the nine *domains*. *Number - Number and Place Value* contains the highest proportion of focus skills (42%) and *Measurement* the lowest proportion (22%). The high proportion of *focus skills* in *Number - Number and Place Value* reflects the number of fundamental concepts contained within this domain, which are essential for pupils to master in order to progress further in mathematics.

4.3 Development of domain expectation statements for each year group

Following on from the completion of *learning progressions,* year level *domain expectation statements* were developed. These statements comprise a descriptive summary of the typical abilities pupils should demonstrate and the typical activities pupils should complete within each year group of each domain.

These statements give teachers an idea of the important developments within a domain, for each year group. An example of a set of mathematics *year level domain expectations* is provided in Appendix C.

5 Mapping the NC-based Learning Progressions to STAR assessment skills

Following the development of the NC-based *learning progressions*, the next stage was to map each *skill statement* to Renaissance Learning's *STAR skills*. The purpose of this stage in the development process was to identify how and where Renaissance Learning's existing extensive bank of mathematics test questions could be used in the context of the new National Curriculum in England. This was a considerable task given that, for mathematics, nearly 1000 *skill statements* had been developed and there were a total of 743 Renaissance Learning *STAR maths skills* to map them to.

The mapping process is summarised in the flow diagram below.

Two researchers collaboratively and systematically worked through the *STAR skill statements* one by one grouped according to US *domain*

Each *STAR skill* was compared to the NC-based *skill statements* from the equivalent domain. For example, STAR skills for *Number and Operations – fractions* were compared with NC-based *skill statements for* Number - fractions (including decimals and percentages). Although there was simillarilty between the domains, some *skill statements* required matching to alternative *domains*.

Similarities between the NC-based *skill statements* and the *STAR skills* were identified through comparison and scrutiny of the individual statements. The *STAR skills* that most closely aligned to each NC-based *skill statement* were then matched up. For example, the NC-based *skill statement* 'Calculate and interpret the mean as an average' was mapped to the *STAR skill* 'Determine the mean of a set of whole number data.' *Some STAR skill's* were mapped to more than one NC-based *skill statement* and vice versa. There were also skill statements from both countries that did not have equivalents.

Once all the *skill statements* for a *domain* had been mapped, the mapping underwent rigorous review, firstly by additional subject specialist Research Associates and then by Renaissance Learning.

Amendments were made on the basis of the feedback given by the Research Associates and Renaissance Learning in order to produce the final mapping of the NC-based *skill statements* to *STAR skills.*

It was decided that the mapping should be completed by two members of the NFER maths team as opposed to being the responsibility of one individual. This was to facilitate discussion about the degree of similarity and alignment between the NC-based *skill statements* and *STAR skills* and to provide the opportunity for consensus to be reached about the most appropriate *STAR skill(s)* to match to each *learning progression skill statement*. It also enabled researchers with different *domain* expertise to input where most needed as, in some cases, the *STAR skills* did not map to NC-based *skill statements* from the equivalent *domains*. For example, the *skill statement* 'Solve percentage problems using knowledge of fractional equivalents*'* is situated within the *Ratio, Proportion and Rates of Change domain* whereas the *STAR skill* it is matched to ('Solve proportion involving decimals*')* falls within the US *Algebra domain*.

In many cases there was no one-to-one correspondence between NC-based *skill statements* and *STAR skills,* as might be expected considering the disparity in the number of statements and the different focuses of the English and US curricula. Often, a single *skill statement* was mapped to multiple *STAR skills*. This ensured that the full breadth of the NC-based *skill statements* were covered by the content of the *STAR skills* assigned to them. For example the *skill statement,* 'Use fractions, decimals and percentages to express probabilities*'*, was matched to two *STAR skills*, 'Determine the probability of a single event*'* and 'Determine the probability of independent events.' Similarly, single *STAR skills* were frequently matched with several *skill statements*.

Once the mapping had been completed, areas of the new National Curriculum that could not be assessed by existing STAR questions were highlighted to Renaissance Learning so that they could begin the process of generating additional questions to extend their existing STAR Maths bank.

Challenges in the mapping process

A number of challenges were encountered in the mapping process, many of which were common to both the mathematics and reading mapping exercises. The first issue related to the different terminology and phraseology used in England and the USA to describe the same mathematical concept. For example in the USA a 'pie chart' is referred to as a 'circle graph'. In order to overcome this issue, key mathematical terms used in the NC-based *skill statements* and *STAR skills* were reviewed. This established whether differences between them reflected genuine differences in mathematical content or simply differences in vocabulary.

The second issue encountered was a mismatch in year / grade equivalence. In many cases a *STAR skill* mapped well to a NC-based *skill statement* with perhaps only a difference of one grade or year. In some instances, however, the differences were much greater. Generally, a US *STAR skill* was only matched to a NC-based *skill statement* if it was assigned to the same academic year or one year either way.

A related issue occurred in cases where multiple *STAR skills* from different grades were mapped to a single *skill statement*. It was agreed with Renaissance Learning that it would be acceptable to have multiple *STAR skills* from different grades mapped to the same *skill statement* if this made sense in terms of the mathematics curriculum for England.

Overall level of matching

Of the 747 *STAR mathematics skills*, 641 (86%) were matched to a NC-based *skill statement*. Of the 106 STAR maths skills which could not be mapped to the NC-based *learning progressions,* the majority of these skills were associated with grades 10 and 11 (years 11 and 12). The two main reasons why STAR skills could not be mapped to NC-based *learning progression skill statements* were:

1. The *STAR skills* covered maths content which is only included in A level mathematics courses. For example, the *STAR skill* ‘Raise imaginary numbers (i) to powers’ includes the concept of imaginary numbers which is not taught in England until A level.
2. The *STAR skills* covered maths content which does not feature in the National Curriculum in England. For example, the *STAR skill* ‘Convert between inches, feet and yards’ does not align to any *skill statements.* In the National Curriculum, pupils are required to convert between different metric units, and to know approximate equivalents between metric units and common imperial units, but are never asked to convert one imperial unit to another.

6 Teachable order

All of the mathematics *skill statements* within each year have been placed in a *teachable order.* This presents the *skill statements* in a sequence in which they could be taught in the classroom. This is a suggested sequence and it is appreciated that there are many alternatives. However, the *teachable order* provides a planning aid which teachers can draw on and its creation was essential in determining the *prerequisite skills* (see Chapter 7) for each *focus skill.*

Within mathematics, some *skill statements* must necessarily be taught before others; for example, pupils will not be able to appreciate that addition is commutative if they have not mastered basic addition skills. However, other *skill statements* have no such dependency; children must be taught basic addition skills and to recognise a square, but mastery of one is not dependent on the other. Teachers may also consider other factors, such as creating variation for their pupils, when deciding the best teaching order. Some of the judgments made when devising the *teachable order* were subjective and, for this reason, it is impossible to develop a definitive sequence. Rather, the *teachable order* presented is just one possibility, and should not be considered prescriptive.

The *skill statements* that must be sequenced in a particular order generally appear in the same *domain* as each other, for example, *skill statements* from the *Geometry domain* are most likely to require prior learning defined by other *Geometry skill statements*. There is less dependency across the different *domains*. For this reason, the initial task in creating the *teachable order* was to sequence the *skill statements* within each *domain*. This was done by considering the *skill statements* one year at a time and ordering all the *skill statements* within the *domains* in that year. Having developed a partial skeleton of the *teachable order*, it became clear that some *domains* contain more *skill statements* that are necessary prerequisites for later learning than others. The *domains* were therefore ordered to reflect this.

The final step in the development of the Reception through to year 6 *teachable orders* required the different *domains* in each year to be amalgamated. The *skill statements* in each *domain* were grouped into teachable chunks which could form topics taught over a week or two. These groups of *skill statements* were slotted together to form an appropriate *teachable order*, taking into consideration the overall devised order of the *domains*, the need to create variety and interest for the pupils, a desire to intersperse difficult topics with those that are more easily mastered but yet still maintain an increasing overall demand throughout each year and finally, to provide a cyclical model of learning that would allow pupils to return regularly to topics to recap before extending their knowledge. This resulted in a final *teachable order* in which the sequencing within each *domain* is strictly adhered to while the *domains* are interwoven. Those *domains* considered to contain more *skill statements* on which future learning depends appear mostly in the first half of the *teachable order* but they also have some representation throughout the year.

The National Curriculum for Reception through to year 6 specifies the knowledge and skills that pupils must be taught on a yearly basis. The clear association between the *skill statements* and the year in which they should be taught made the creation of a *teachable order* for each school year an obvious choice. However, the National Curriculum is organised differently for year 7 and above. It presents the knowledge and skills that pupils must master within each key stage rather than in each year. Secondary school teachers therefore have more flexibility when planning their teaching. Key stage 3 material can be taught in any of years 7, 8 and 9, while key stage 4 material can be taught in either year 10 or year 11. As discussed previously, the sequencing of the *skill statement* topic groups for the primary years is somewhat subjective. If the secondary material was also ordered within years, the need to assign a year to each of the *skill statements* in key stage 3 or 4 would cause even greater subjectivity. For this reason, the *teachable order* for key stages 3 and 4 presents all the *skill statements* in their *domains* without integration. Both the order of the *domains* and the sequence of the *skill statements* within each *domain* are strictly adhered to. Whilst it is appreciated that this does not demonstrate an appropriate final *teachable order*, it allows teachers to fully utilise the dependency information when devising the best teaching scheme for their pupils.

The mathematics *teachable order* indicates the order in which the *skill statements* should first be taught to pupils, as opposed to when the teaching of the skill is ‘finished’. Therefore, core skills such as being able to add are presented towards the beginning of the *teachable order* despite the fact that they continue to be reinforced throughout the year.

The *teachable order* was developed by four qualified teachers, each having responsibility for the key stage in which their expertise lies. Although the initial *teachable order* was devised individually, all were reviewed by a second teacher and then fine-tuned during a period of consultation. A final review was carried out by Renaissance Learning before approval.

7 Teacher table information

Teacher tables were created for all of the *skill statements* that had been identified as *focus skills.* This phase of work was delegated amongst the team so that each researcher had responsibility for drafting the *teacher tables* for the *domains* which they had originally authored during the development of *learning progressions.* The construction of the *teacher tables* followed the development of the *teachable order* as the *teachable order* was used as a basis for identifying the prerequisite skills. It also enabled researchers to refer to all the *skill statements* across all the *domains* where necessary. Table 3 shows the type and purpose of the additional information in the *teacher tables*, along with an example of content from a *Geometry teacher table*.

Table 3. Information in each teacher table and a Geometry exemplar

Column heading	Type of information and purpose	Example Focus skill: Derive formulae to calculate the area of triangles, parallelograms and trapezia
Working mathematically	for key stages 3 and 4 only *Working mathematically* skills from the National Curriculum developed through this *focus skill* (skill statement code only)	KS3 WM.DF.2; KS3 WM.DF.5; KS3 WM.RM.2; KS3 WM.RM.5; KS3 WM.SP.1
New terminology, concepts and skills needed for this focus skill	new terminology, concepts and skills which the student develops through this *focus skill*	area; calculate areas of triangles; calculate areas of parallelograms; calculate areas of trapezium
EAL Support	additional support that may be given to pupils with EAL so that they can understand and participate in the study of this *focus skill*	Review the terms *parallelogram and trapezium*. Verify that students understand the meaning of the mathematical term *derive a formula*.
Prerequisite terminology, concepts and skills	other *terminology, concepts and skills* which must be understood by pupils before teaching this *focus skill*	parallelogram; trapezium; derive formulae
Prerequisite skill statements	*skill statements* which must be understood by pupils before teaching this *focus skill*	Derive formulae to calculate the perimeter of triangles, parallelograms and trapezium
Prerequisite skill ID	pre-determined code used to classify the *skill statement(s)* identified as a prerequisite skill statement above	M_10525

Work on the teacher tables was begun by considering the *prerequisite terminology, concepts and skills* column*.* Researchers identified the terminology, concepts and skills which supported or underpinned a specific focus skill. They then identified the *prerequisite skill statements* and the corresponding *IDs* which most closely matched the terminology, concepts and skills identified (i.e. the skills that underpin the focus skill). More than one *prerequisite skill statement* could be included but only *skill statements* up to two years prior to each *focus skill* were considered. The *teachable order* was an important resource in establishing the *prerequisites* for each *focus skill* because only those *skill statements* which came before it in the *teachable order* could be considered as possible *prerequisite skill statements*.

The next field to be completed was *new terminology, concepts and skills*. Researchers compared the *prerequisite terminology, concepts and skills* with the *focus skill* in order to decide what new terminology, concepts and skills the students were acquiring in this *focus skill*. Despite endeavouring to extract specific new terminology, concepts and skills from the *skill statement,* and not merely repeat the *skill statement* itself, this was not always possible, as often the entire *skill statement* described the new skill. Although it is described here as a linear process, in reality, the development of the *teacher tables* often went through several iterations and gradual refinement with comparisons between the *prerequisite* and *new terminology, concepts and skills*.

For each *focus skill* in key stages 3 and 4, the related *working mathematically* statements were also identified. *Working mathematically* skills are incorporated within the programme of study in key stages 1 and 2, but are given as separate statements at key stages 3 and 4. Consequently, only *focus skills* in key stages 3 and 4 include a *working mathematically* component. The *working mathematically* skills are overarching statements and more than one *working mathematically* statement is often relevant to a *focus skill*. The *working mathematically* statements are long so, to minimise space, they have been allocated a reference number which teachers can look up on a centralised list. For example KS3 WM.DF.2 represents the second bullet in the *Develop fluency* section of the *Working mathematically* statements in the key stage 3 National Curriculum.

To populate the *EAL support* field, key vocabulary words, important instruction commands words and key conceptual terminology were identified. These were judged to be the important terms required to approach that stage of learning on an equal footing with native English speakers. Completion of this field centred on supporting the pupils' language, not his or her mathematical ability. The words and terminology identified often coincided with, or were similar to, the prerequisite information provided in the *teacher tables*, which covered the previous stages of learning. An EAL specialist completed and reviewed the work on this field.

Once the *teacher table* for each *domain* was completed, it was reviewed by external consultants who checked for accurate and appropriate content. Finally another member of the research team checked for consistency within and between tables in addition to a review by Renaissance Learning.

8 Conclusion

Commissioned by Renaissance Learning, NFER has developed: a *hierarchy* that reflects how mathematics is conceptualised in the new National Curriculum in England, *learning progressions*, individual *skill statements*, *teacher tables* and suggested sequences of teaching the skills in the *teachable order* from Reception to year 11. *Learning progressions* provide a ‘roadmap’ to guide teaching and learning and to ensure that pupils develop their subject knowledge in a coherent and consistent way.

It is hoped that these *learning progressions* will help to support schools to develop assessment frameworks to track pupil progress, following the abolition of National Curriculum levels.

The mapping of the *learning progressions* to the skills assessed within *STAR Maths* demonstrates the potential for such assessments to support teachers in monitoring progress against the programme of study for mathematics. The assessments can help to indicate a pupil’s current level of knowledge and the *learning progressions* then suggest next steps in his or her learning.

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Appendix A: Framework for Learning Progressions – Mathematics

Domain	Skill Area	Age range
Number – number and place value	• Counting • Place value • Negative numbers • Roman numerals • Problem solving using number and place value	*Early years – Year 10*
Number – arithmetic operations	• Arithmetic methods • Multiples, factors and prime numbers • Powers and roots • Problem solving using arithmetic	*Early years – Year 11*
Number - fractions (including decimals and percentages)	• Fraction concepts and operations • Decimal concepts and operations • Percentages • Problem solving using decimals, fractions and percentages	*Year 1 – Year 10*
Ratio, proportion and rates of change	• Understanding and finding ratios • Using ratio in shapes and measurement	*Year 6 – Year 11*
Algebra	• Expressions and equations • Functions • Problem solving using algebra	*Year 6 – Year 11*
Measurement	• Conducting and recording measurements • Problem solving using measurements	*Year 1 – Year 6*
Geometry	• Identifying, describing and sorting geometric shapes • Constructing geometric shapes • Translation, rotation, and coordinate geometry • Understanding, describing and constructing angles, segments & lines • Circle theorem • Problem solving using geometry	*Early years – Year 11*
Probability	• Describing and finding probabilities • Representing probabilities diagrammatically	*Year 7 – Year 11*
Statistics	• Data representation • Data analysis	*Year 2 – Year 11*

Organisation of National Curriculum into Skill Areas within the nine domains

Appendix B: Learning Progressions for Measurement (Years 4-6) (Example)

Highlighted cells denote *skill statements* added to bridge gaps.

Year	National Curriculum statement	*Skill statement*
4	measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres	Measure the perimeter of a rectilinear figure (squares and other shapes) in centimetres and metres
		Calculate the perimeter of a rectilinear figure (squares and other shapes) in centimetres and metres
4	find the area of rectilinear shapes by counting squares	Understand the concept of area
		Find the area of simple shapes
		Find the area of rectilinear shapes by counting squares
4	convert between different units of measurement [for example, kilometre to metre; hour to minute]	Convert between different units of measurement
4	estimate, compare and calculate different measures, including money in pounds and pence	Compare different measures, including money in pounds and pence
		Estimate different measures, including money in pounds and pence
		Calculate different measures, including money in pounds and pence
4	read, write and convert time between analogue and digital 12- and 24-hour clocks	Read, write and convert time between analogue and digital 12-hour clocks
		Read, write and convert time between analogue and digital 24-hour clocks
4	solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days	Know the relationships between different units of time
		Solve problems involving converting from hours to minutes
		Solve problems involving converting from minutes to seconds
		Solve problems involving converting from years to months
		Solve problems involving converting from weeks to days

Year	National Curriculum statement	*Skill statement*
5	measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres	Measure the perimeter of composite rectilinear shapes in centimetres and metres
		Understand the concept of composite shapes and be able to visually divide a composite shape into smaller shapes
		Calculate the perimeter of composite rectilinear shapes in centimetres and metres
5	calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes	Know areas you would measure in different units of measurement (mm^2, cm^2, m^2)
		Compare the area of rectangles and squares
		Calculate the area of rectangles and squares
		Estimate the area of irregular shapes
5	convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)	Convert between different units of metric measure
5	understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints	Understand approximate equivalences between metric units and common imperial units such as inches, pounds and pints
		Use approximate equivalences between metric units and common imperial units such as inches, pounds and pints
5	estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]	Estimate volume
		Estimate capacity
5	solve problems involving converting between units of time	Solve problems involving converting between units of time
5	use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.	Solve measurement problems (length, mass, volume, money) using all four operations. Use decimal notation and scaling

Year	National Curriculum statement	*Skill statement*
6	solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate	Solve problems involving the calculation and conversion of units of measure
6	use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places	Convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places
6	convert between miles and kilometres	Convert between miles and kilometres
6	recognise that shapes with the same areas can have different perimeters and vice versa	Recognise that shapes with the same areas can have different perimeters Recognise that shapes with the same perimeters can have different areas.
6	recognise when it is possible to use formulae for area and volume of shapes	Recognise when it is possible to use formulae for area and volume of shapes
6	calculate the area of parallelograms and triangles	Calculate the area of parallelograms and triangles
6	calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3].	Compare volume of cubes and cuboids using standard units Calculate volume of cubes and cuboids using standard units Estimate volume of cubes and cuboids using standard units

Appendix C: Year-Level Expectations for *Statistics*

Reception	n/a
Year 1	n/a
Year 2	Students collect information by voting or sorting, and represent the information by making marks or drawing or placing objects. They construct and interpret tally charts, block diagrams, simple tables with one variable and simple pictograms. Students understand and use appropriate data vocabulary, they make and organise lists of data and they sort and match objects, justifying their decisions. Students answer questions about data samples and ask and answer questions by counting objects in categories, sorting categories by quantity, and totalling and comparing categorical data.
Year 3	Students present and interpret data using bar charts, pictograms and tables. They answer one- and two-step questions using information presented in these formats.
Year 4	Students use appropriate graphical methods to present and interpret discrete and continuous data. They use bar charts, pictograms, tables and other graphs to solve problems involving comparisons, sums and finding differences.
Year 5	Students complete, read and interpret information presented in tables. They use information presented in line graphs to solve problems involving comparisons, sums and finding differences.
Year 6	Students construct and interpret pie charts and line graphs and use these to solve problems. They calculate and interpret the mean as an average and they describe observed distributions of single variables through appropriate graphical representations and measures of central tendency and spread.
Year 7	Students construct and interpret appropriate tables, charts and diagrams for categorical data and vertical line charts for ungrouped and grouped numerical data. They compare and interpret observed distributions of a single variable through appropriate measures of central tendency and spread, and through graphical representations involving discrete, continuous and grouped data.

Year 8 Students interpret appropriate tables for categorical and numerical data. They describe simple mathematical relationships between two variables, in observational and experimental contexts, and illustrate these using scatter graphs.

Year 9 Students calculate statistics and select those most appropriate to the problem.

Year 10 Students construct, interpret and know the appropriate use of diagrams for grouped discrete data and continuous data. They interpret and construct line graphs and tables for time series data. Students use and interpret scatter graphs of bivariate data and recognise that correlation does not indicate causation. They interpret, compare and analyse the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency and spread and through appropriate graphical representation, such as box plots. Students recognise when to use measures of central tendency and they calculate statistics for sets of discrete and continuous data. Students understand the difference between a sample and a population and are able to infer properties of populations and distributions from a sample, whilst knowing the limitations of sampling.

Year 11 Students apply statistics to describe a population. They draw on lines of best fit and make predictions from scatter graphs. They interpolate and extrapolate apparent trends based on scatter graphs, whilst knowing the dangers of doing so.

NFER provides evidence for excellence through its independence and insights, the breadth of its work, its connections, and a focus on outcomes.

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